



Development of a Low-Alloy Steel Stress-Strain Curve Simulation Model Using the Ramberg-Osgood Approach

Yuda Perdana Kusuma¹, Nasrullah^{2*}, Hooi Peng Lim³, Muchlisinalahuddin⁴, Muhammad Rabiu Abbas⁵

¹Asosiasi Diseminasi Rekayasa Dan Inovasi Teknologi, Padang, Indonesia

²Department of Mechanical Engineering, Politeknik Negeri Padang, Padang, Indonesia

³Department of Mechanical Engineering, Politeknik Ibrahim Sultan, Johor, Malaysia

⁴Department of Mechanical Engineering, Universitas Muhammadiyah Sumatera Barat, Bukittinggi, Indonesia

⁵Department of Mechanical Engineering, The Federal University of Transportation Daura, Katsina State, Nigeria

*Corresponding author: Nasrullah, 9nasrullah@gmail.com.

ABSTRACT

The stress-strain curve is essential information in the analysis of a material's mechanical behavior. However, obtaining the curve directly requires specialized testing that is not always available. Standard tensile test parameters such as yield strength, tensile strength, elongation, and reduction in area are commonly available from routine material testing. This study addresses this limitation by developing a low-alloy steel stress-strain curve simulation model based on the Ramberg-Osgood equation using these four mechanical parameters as the primary inputs. The main parameter in this model is the strain hardening exponent (n), which is calculated directly from the available mechanical data. The calculated n values ranged from 1.992 to 38.211, with an average value of 14.17, which is consistent with the general characteristics of low-alloy steels. The simulated curves exhibited profiles consistent with the behavior of ductile metals, where each specimen produced different plastic deformation characteristics according to its respective mechanical properties. Internal validation of the simulation demonstrated that the simulated curves showed complete agreement with the mechanical input parameters, with no deviation across all analyzed samples. The results indicate that low-alloy steel stress-strain curves can be accurately reconstructed using only conventional tensile test data. Therefore, the developed model has the potential to serve as a practical solution for generating stress-strain curves when complete experimental curve data are unavailable but required for further engineering analysis.

KEY WORDS : *Low-alloy steel, stress-strain curve, ramberg-Osgood, strain hardening exponent, material simulation.*

1. INTRODUCTION

Low-alloy steel is one of the most widely used materials in the manufacturing, construction, and machinery industries due to its ability to provide a good combination of mechanical strength, toughness, and ease of fabrication. In the design and analysis of steel components, understanding the mechanical behavior of the material is a crucial aspect [1],[2],[3]. One of the most comprehensive representations of this behavior is the stress-strain curve, which describes the material response to loading continuously from the elastic region to the plastic region within a single diagram. Stress-strain curves play an important role in various modern engineering analyses, including finite element simulations, structural failure analysis, and component fatigue life prediction [4]. Several international engineering standards, including the ASME Boiler and Pressure Vessel Code and Eurocode 3, also require stress-strain curve data

as input for elastoplastic material analysis [5],[6]. Therefore, the availability of accurate stress-strain curves is not only important for academic purposes but also has significant practical value in industrial applications. However, obtaining a complete stress-strain curve requires tensile testing using an extensometer capable of continuously recording strain throughout the loading process. In practice, such equipment is not always available, particularly in routine industrial testing facilities that generally only provide basic mechanical parameters such as yield strength, tensile strength, elongation, and reduction in area. Although a large amount of tensile test data are available in the literature, complete stress-strain curve information is not readily available. This gap becomes a significant limitation when such data are required as input for further engineering analysis.

These limitations have encouraged the development of various mathematical approaches to reconstruct stress-strain curves from standard tensile test parameters. One of the most widely adopted approaches is the Ramberg-Osgood equation, introduced in 1943. This model is capable of representing the transition from elastic to plastic behavior continuously using only a few key mechanical parameters [7],[8]. In addition, previous studies [9] demonstrated that Ramberg-Osgood curve parameters can be reliably estimated using yield strength and ultimate tensile strength for steel materials with or without Lüders strain. Another study [10] also reported that monotonic tensile test parameters are sufficient to generate stress-strain curve estimations with adequate accuracy for engineering analysis purposes.

However, most previous studies have primarily focused on model validation using actual experimental stress-strain curve data with a relatively limited number of specimens. To date, limited research has investigated the systematic application of the Ramberg-Osgood model under conditions where complete experimental curve data are unavailable, despite such conditions being commonly encountered in industrial material databases and routine laboratory testing results. This research gap forms the basis and motivation of the present study.

Based on these considerations, this study aims to develop a low-alloy steel stress-strain curve simulation model based on the Ramberg-Osgood equation using standard tensile test parameters as the primary inputs. The developed model is expected to serve as a practical tool for reconstructing and visualizing stress-strain curves accurately without requiring additional complex experimental testing.

2. METHODOLOGY

This study employed a Research and Development (R&D) method with a numerical simulation development approach based on the Ramberg-Osgood mathematical model to develop a stress-strain curve simulation system capable of interactively visualizing the mechanical behavior of materials. This approach has been widely applied in previous material engineering studies [11],[12],[13] as a semi-empirical model for representing the continuous elastic-plastic behavior of metallic materials within a single equation. The overall stages of the proposed stress-strain curve visualization model development are illustrated in Figure 1.

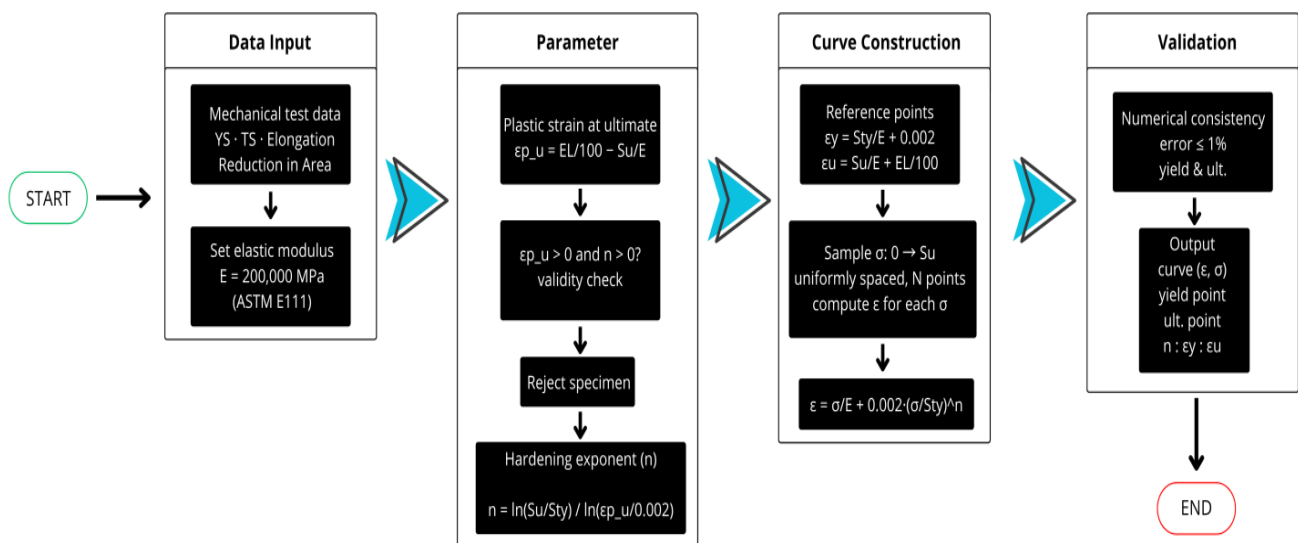


Figure 1. Stages of the stress-strain curve visualization model development.

1. Data and Materials

The study used low-alloy steel tensile test data obtained from a public dataset platform, Kaggle [14]. The dataset consisted of four primary mechanical parameters: Yield Strength (YS, MPa), Tensile Strength (TS, MPa),

Elongation (eL, %), and Reduction in Area (RA, %). However, the dataset did not provide information regarding the elastic modulus (E). Therefore, the elastic modulus was assumed to be 200 GPa based on the standard reference value for steel materials according to ASTM E111 [15]. This value was assumed because the elastic modulus of steel is generally considered relatively constant and is not significantly affected by minor alloy composition variations.

2. Ramberg-Osgood Model

The stress-strain curves were reconstructed using the Ramberg-Osgood equation [16]:

$$\varepsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{\sigma_{ty}} \right)^n \quad (1)$$

where ε represents strain, σ represents stress, E is the elastic modulus, σ_{ty} denotes the yield strength, and n is the strain hardening exponent that characterizes the strain hardening behavior of the material.

The main reference points of the curve were determined based on two conditions, namely the yield point and the ultimate point [17],[18]. The yield point was defined using the 0.2% offset method:

$$\varepsilon_y = \frac{\sigma_{ty}}{E} + 0.002 \quad (2)$$

Meanwhile, the ultimate point was determined using:

$$\varepsilon_u = \frac{\sigma_u}{E} + \frac{eL}{100} \quad (3)$$

The strain hardening exponent (n) was then calculated numerically using the tensile test mechanical data [19]. The first step involved calculating the plastic strain at the ultimate condition using:

$$\varepsilon_{p_u} = \frac{eL}{100} - \frac{\sigma_u}{E} \quad (4)$$

Subsequently, the value of n was calculated using the following relationship:

$$n = \frac{\ln\left(\frac{\varepsilon_{p_u}}{0.002}\right)}{\ln\left(\frac{\sigma_u}{\sigma_{ty}}\right)} \quad (5)$$

Data producing $\varepsilon_{p_u} \leq 0$ or $n \leq 0$ were considered invalid and excluded from the analysis because they did not satisfy the characteristics of plastic deformation behavior [9].

3. Stress-Strain Curve Construction

The stress-strain curves were constructed through a uniform stress sampling process from 0 up to the ultimate tensile strength (σ_u) using N data points. For each stress value, the total strain (ε) was calculated using Equation (1) with the strain hardening exponent (n) obtained from Equation (5) and an elastic modulus value of 200,000 MPa. This process generated pairs of (ε, σ) data points that formed a continuous stress-strain curve from the unloaded condition to the maximum stress state of the material. Figure 2 illustrates the general form of the stress-strain curve generated by the Ramberg-Osgood model. The curve consists of two main regions: a linear elastic region with a slope corresponding to the elastic modulus (E), and a nonlinear plastic region that develops after the material passes the yield point. The transition between these two regions is defined by the yield point determined using the 0.2% offset method. The curve terminates at the ultimate point, which represents the maximum stress sustained by the material before failure.

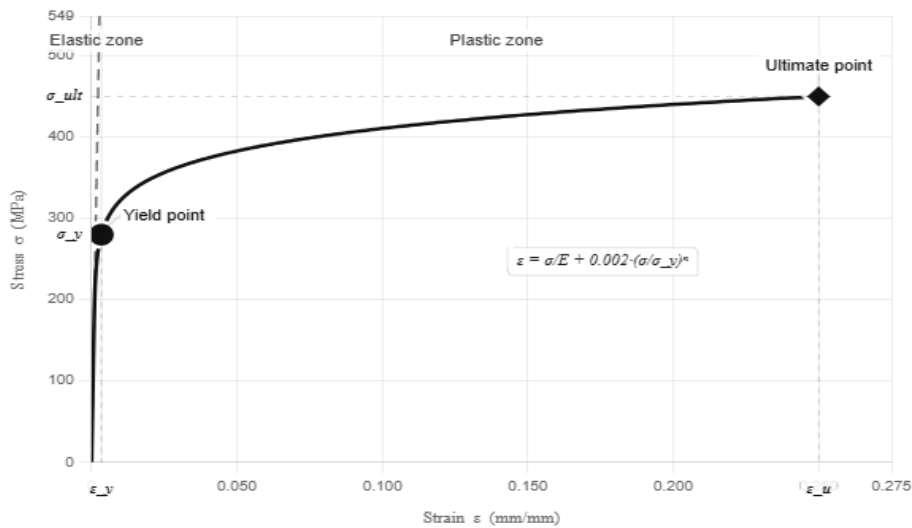


Figure 2. Schematic of the Ramberg-Osgood stress-strain curve. Adapted from Ramberg & Osgood [16].

4. Stress-Strain Curve Model Validation

Validation was performed to evaluate the numerical consistency between the simulated stress-strain curves and the mechanical input parameters used in the model. Two main validation criteria were applied.

- a. The first criterion was the accuracy of the yield point, where the stress value on the simulated curve at the strain condition of ϵ_y was required to match the yield strength value (S_{ty}) with a maximum relative error tolerance of 1%.
- b. The second criterion was the accuracy of the ultimate point, where the stress value on the simulated curve at the strain condition of ϵ_u was required to match the ultimate tensile strength value (S_u) with a maximum relative error tolerance of 1%.

The simulation model was considered valid when both validation criteria were satisfied for all analyzed samples.

3. RESULTS AND DISCUSSION

The results of this study focused on the development of a low-alloy steel stress-strain curve simulation model using the Ramberg-Osgood approach. The analysis began with a statistical characterization of the dataset to examine the distribution and variation of the mechanical parameters used as model inputs. Subsequently, the strain hardening exponent (n) was calculated as the primary parameter governing the plastic deformation behavior of the material, followed by an analysis of its distribution and relationship with other mechanical properties. The simulated stress-strain curves were then visualized to demonstrate the capability of the proposed model to continuously represent the elastic and plastic behavior of low-alloy steels with varying mechanical characteristics. In addition, internal validation was conducted to evaluate the numerical consistency between the simulated curves and the mechanical input parameters used during the modeling process.

1. Mechanical Property Characteristics of Low-Alloy Steel

The dataset used in this study consisted of 915 low-alloy steel tensile test samples with four primary mechanical parameters: Yield Strength (YS), Tensile Strength (TS), Elongation (eL), and Reduction in Area (RA). Statistical summaries and visualizations of all mechanical parameters are presented in Table 1 and further illustrated in Figure 3.

Table 1. Statistical summary of low-alloy steel mechanical properties.

Parameter	Min	Max	Mean	Std. Deviation	Median
Yield Strength (MPa)	27	690	328.22	131.65	290
Tensile Strength (MPa)	162	830	489.7	125.97	479
Elongation (%)	10	78	26.79	8.81	26
Reduction in Area (%)	18	94	70.21	12.39	71

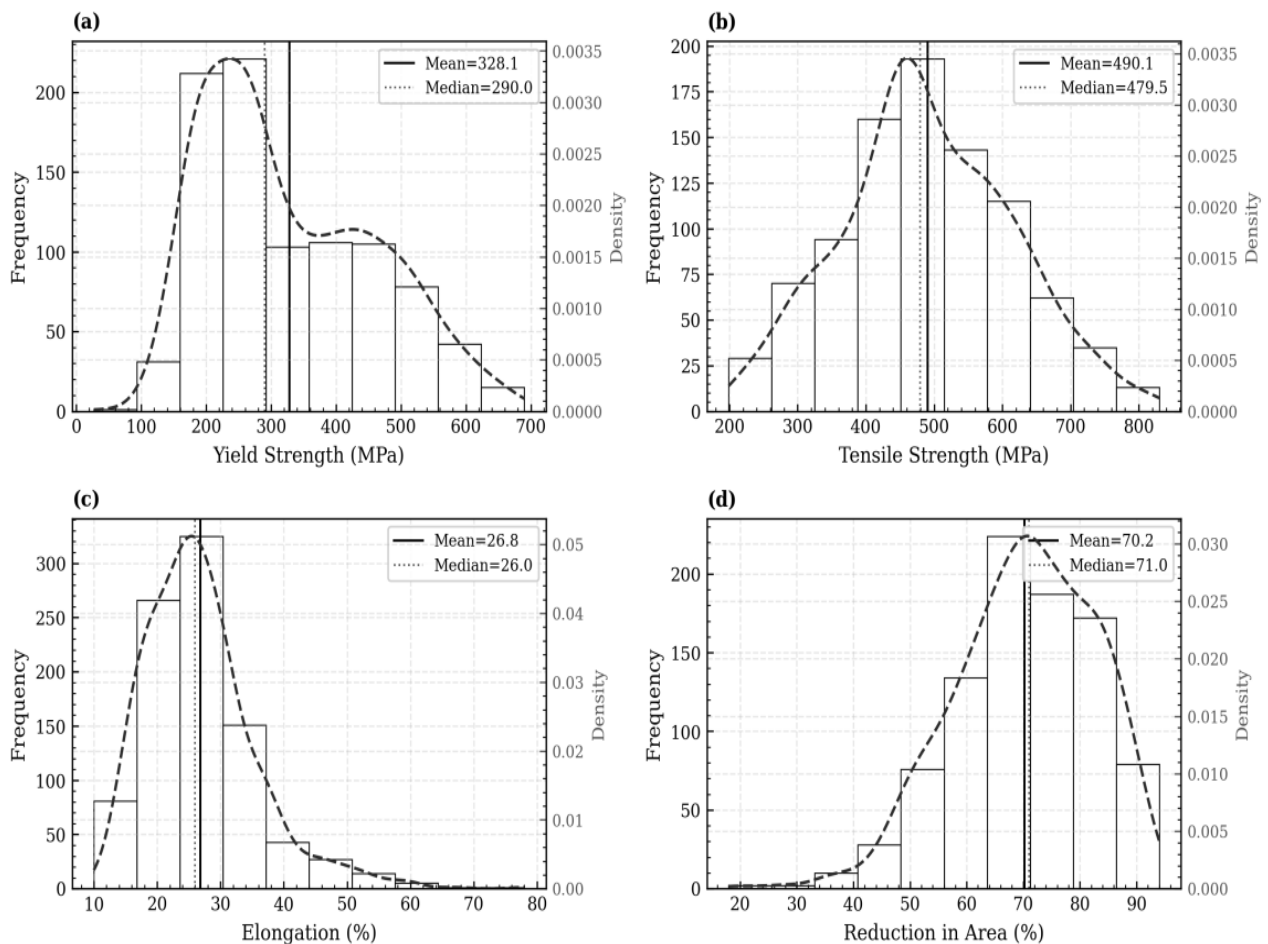


Figure 3. Distribution of low-alloy steel mechanical properties.

The distribution of the four mechanical parameters from the 915 low-alloy steel samples exhibited considerable variability. Yield strength showed an average value of 328.22 MPa and a median of 290.0 MPa. The difference between these two values indicates that most samples were concentrated within the low to medium strength range, while a smaller number of high-strength samples shifted the average toward higher values. A similar trend was observed for tensile strength, with an average of 489.7 MPa and a median of 479 MPa, although the distribution appeared more uniform and symmetric.

Elongation exhibited a different distribution pattern, where most samples were concentrated within the 20–30% range, with an average of 26.8% and a median of 26.0%. However, several samples showed very high elongation values reaching up to 78%, which may be associated with testing conditions at elevated temperatures. Reduction in area displayed the most uniform distribution among all parameters, with an average of 70.2% and a median of 71.0%, indicating that most samples possessed relatively consistent ductility characteristics representative of low-alloy steel materials.

2. Calculation of Ramberg-Osgood Model Parameters

The strain hardening exponent was calculated for all 914 valid samples using Equation (4) and Equation (5). The distribution of the calculated n values is presented in Figure 4. The obtained n values ranged from 1.992 to 38.211, with an average value of 14.17 and a median of 13.07.

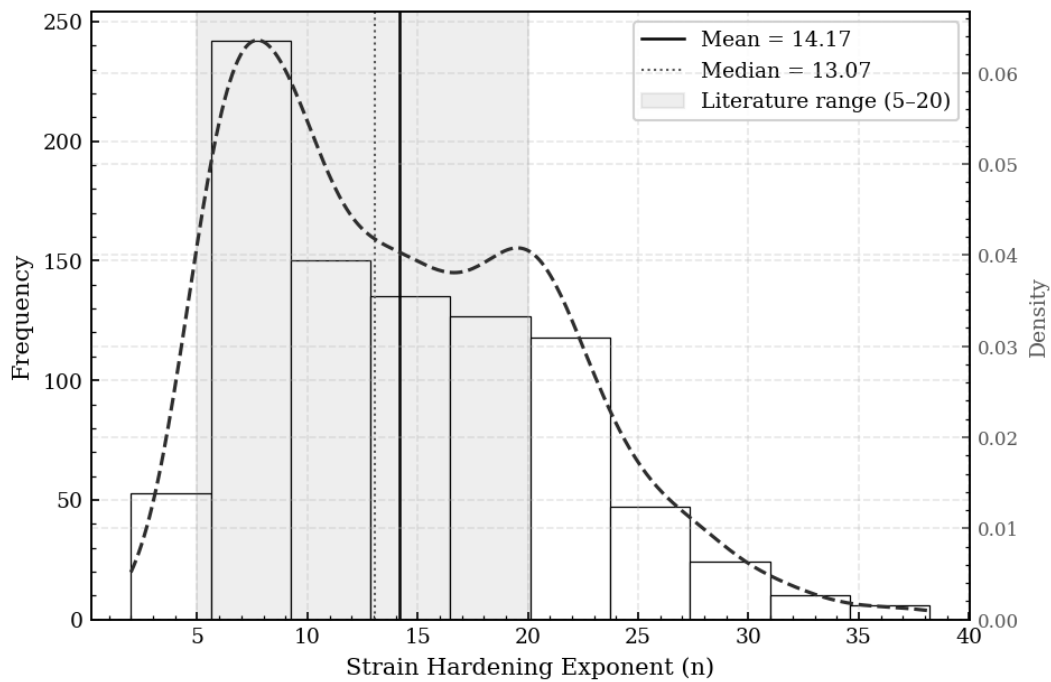


Figure 4. Distribution of strain hardening exponent (n) values.

As shown in Figure 4, most samples produced n values within the range of 5 to 20, which is commonly reported for low-alloy steels. These findings are consistent with previous work [19], which estimated Ramberg-Osgood parameters for ferritic steels using yield strength and ultimate tensile strength and reported a similar range of n values. In addition, another study [20] on ASTM A302-B steel demonstrated that the strain hardening exponent of low-alloy steel remains relatively stable under varying temperature conditions, indicating that n represents an intrinsic material characteristic rather than merely a testing-dependent parameter. The close agreement between the average and median values also indicates that the distribution of n was relatively balanced, although a small number of samples exhibited values higher than the overall average.

In addition to the n parameter, the yield strain (ϵ_y) and ultimate strain (ϵ_u) were also calculated for each sample. The calculated ϵ_y values ranged from 0.00214 to 0.00545, with an average of 0.00364, representing the elastic strain limit of the material. Meanwhile, the ϵ_u values ranged from 0.1030 to 0.7811, with an average of 0.2704. The significantly wider range of ϵ_u compared to ϵ_y indicates that most deformation occurring in low-alloy steel is dominated by plastic deformation.

This behavior is consistent with previous findings [21], which explained that ductile metallic materials such as steel are capable of undergoing substantial plastic deformation before reaching their maximum stress condition, as reflected by the large difference between elastic strain and total strain at failure. Furthermore, elongation and reduction in area are widely recognized as primary indicators of material ductility, where ductile materials tend to experience extensive plastic deformation prior to fracture. These characteristics explain why the Ramberg-Osgood model is particularly suitable for low-alloy steel, as the equation was specifically developed to represent the elastic-to-plastic transition in metallic materials with well-defined and extended plastic deformation regions [22].

3. Stress-Strain Curve Construction

The stress-strain curves were generated using the Ramberg-Osgood equation. Figure 5 presents the stress-strain curves of three representative specimens selected based on low, medium, and high yield strength values to represent the range of material characteristics within the dataset.

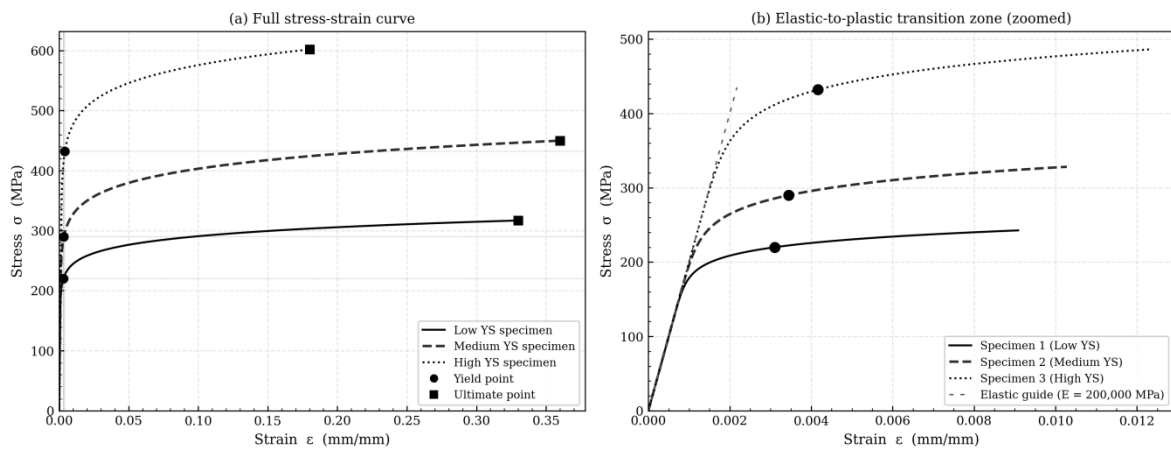


Figure 5. Stress-strain curves of three representative specimens: (a) complete curves from the yield point to the ultimate point; (b) enlarged view of the elastic-to-plastic transition region.

The comparison of the three curves demonstrates patterns consistent with the mechanical behavior of ductile metallic materials, characterized by a linear elastic region at low stress levels followed by a gradually nonlinear plastic region until reaching the maximum stress of each specimen. Specimens with higher yield strength produced curves located at higher stress levels and exhibited greater initial resistance to deformation; however, they also showed shorter plastic deformation regions compared to specimens with lower yield strength. This behavior reflects the typical trade-off between strength and ductility commonly observed in low-alloy steels. Previous studies [18] reported that increasing the strength of steel is generally accompanied by a reduction in its plastic deformation capacity.

The enlarged elastic-to-plastic transition region shows that all three curves originated from the same initial point and followed the elastic slope corresponding to $E=200,000$ MPa before deviating from linearity at different yield points according to their respective yield strength values. The variation in curve curvature within the transition region directly reflects differences in the n values among the specimens, where specimens with higher n values exhibited a sharper transition from the elastic region to the plastic region. These findings are consistent with previous work [16], which stated that the n parameter controls the curvature of the stress-strain curve in the transition region and serves as a quantitative representation of the material strain hardening behavior.

To illustrate the variability of the entire dataset, Figure 6 presents an overlay of stress-strain curves from 150 randomly selected specimens taken from the 914 samples included in the main analysis, together with the median specimen curve used as a reference.

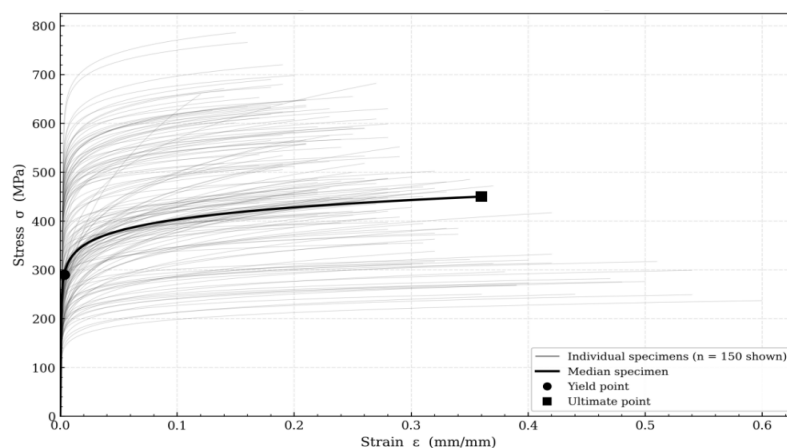


Figure 6. Stress-strain curves of 150 specimens with the median curve as a reference.

As shown in Figure 6, all curves exhibit a generally consistent pattern despite the considerable variation in stress and strain values among the specimens. The largest variation appears in the plastic region, particularly within the ultimate strain range, reflecting differences in ductility among the samples in the dataset. The median specimen curve, with a yield strength of 290 MPa and a tensile strength of 479 MPa, is located near the center of the distribution and represents the typical mechanical characteristics of low-alloy steel within this dataset. The consistency of the curve patterns across all samples confirms that the developed simulation model is capable of generating representative stress-strain curves not only for a single material condition but also across a broad range of mechanical properties.

At this stage, a sensitivity analysis was also conducted to investigate the influence of the n parameter on the shape of the stress-strain curve. The analysis was performed by varying the n value while maintaining the same

specimen properties. The selected specimen corresponded to the median dataset values, namely $YS = 290$ MPa, $TS = 479$ MPa, and $EL = 26\%$, while these parameters were kept constant throughout the analysis. Therefore, any changes observed in the curve shape were solely caused by variations in the n value rather than differences in the material mechanical properties. The results of the sensitivity analysis are presented in Figure 7.

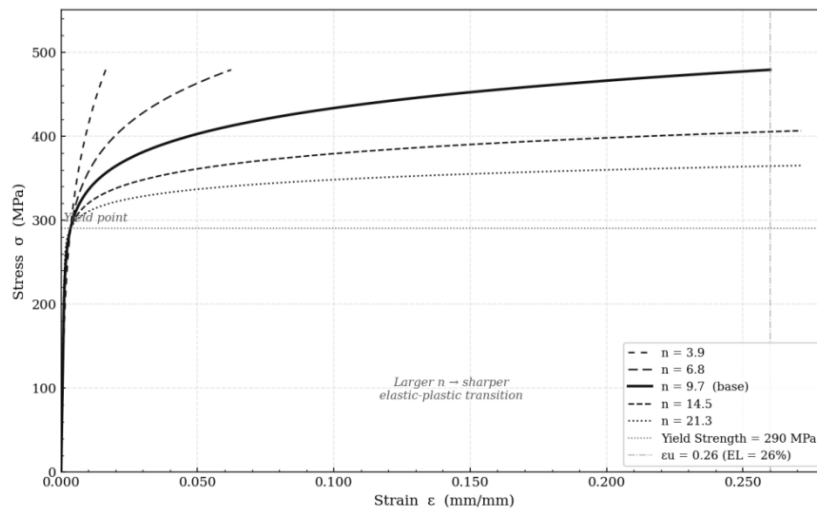


Figure 7. Sensitivity analysis of stress-strain curve shape with respect to variations in n values.

The results indicate that lower n values produce curves that rise more rapidly and sharply after passing the yield point, whereas higher n values generate smoother and more gradual curve transitions. Nevertheless, all curves originate from the same point and terminate at the same ultimate stress because the YS and TS values remain unchanged. These findings demonstrate that the n parameter specifically governs the strain hardening behavior within the plastic region without affecting the elastic limit or the ultimate stress of the material.

4. Stress-Strain Curve Model Validation

Model validation was performed by comparing the stress values at the simulated yield and ultimate points with the mechanical input data used during the curve construction process, using a maximum relative error tolerance of 1%. For visualization purposes, 10 representative specimens were selected to cover the full range of yield strength values within the dataset, from 27 MPa to 690 MPa, allowing the validation results to represent the overall variation of the data. The validation results are presented in Figure 8. The validation results demonstrate that the proposed model achieved excellent agreement between the simulated curves and the tensile test input data for all analyzed samples. Both the mean error and maximum error at the two validation points were 0.0000%, indicating that the simulated curves passed exactly through the yield and ultimate points defined by the mechanical input parameters. These findings confirm that the Ramberg-Osgood approach was able to maintain numerical consistency throughout the stress-strain curve reconstruction process.

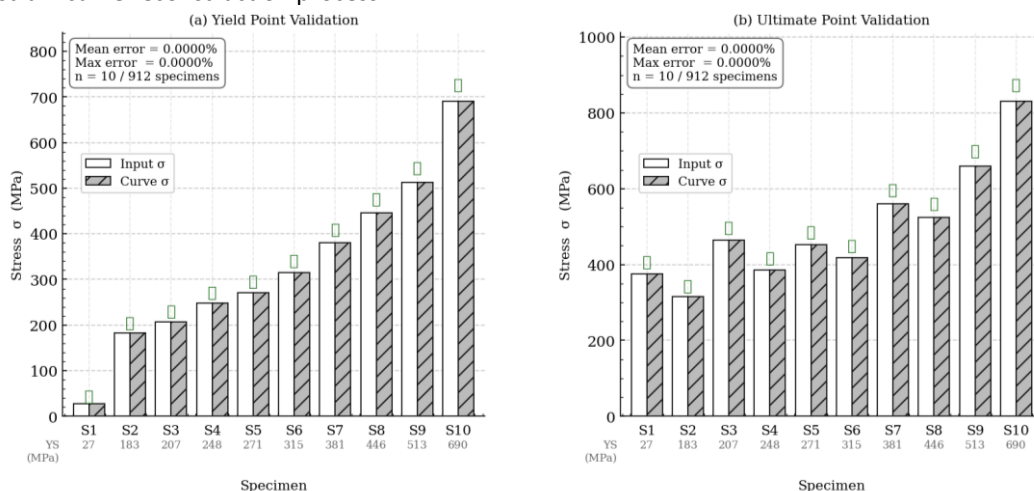


Figure 8. Comparison between input stress values and simulated curve stress values at the yield point (a) and ultimate point (b) for 10 selected specimens.

The results of this study are consistent with previous findings [9], which reported that the Ramberg-Osgood approach based on standard mechanical parameters can reliably represent the elastic-plastic behavior of steel when the strain hardening exponent (n) is determined consistently according to the material boundary conditions. Another study [10] also demonstrated that monotonic tensile test parameters are sufficient to estimate steel stress-strain curves with high accuracy. Overall, the validation results indicate that the developed simulation model is capable of accurately reconstructing low-alloy steel stress-strain curves using only conventional tensile test data, without requiring additional experimental deformation measurements or more complex testing procedures.

4. CONCLUSION

This study successfully developed a low-alloy steel stress-strain curve simulation model based on the Ramberg-Osgood equation using four standard tensile test parameters as inputs, namely yield strength, tensile strength, elongation, and reduction in area. Out of 915 available data samples, 914 satisfied the model validity requirements and were included in all stages of the analysis. The resulting strain hardening exponent (n) values ranged from 1.992 to 38.211, with an average value of 14.17, which is consistent with the characteristics of low-alloy steels reported in previous studies. The generated stress-strain curves exhibited elastic and plastic deformation behaviors consistent with the mechanical characteristics of metallic materials, while variations in curve shape among specimens reflected the diversity of strain hardening behavior within the dataset.

The internal validation results demonstrated that the model achieved excellent numerical consistency at both the yield and ultimate points for all analyzed samples. The findings indicate that low-alloy steel stress-strain curves can be accurately reconstructed using only conventional tensile test data without requiring direct strain curve measurements through extensometer testing. This makes the developed model potentially useful as a practical tool for material behavior analysis, particularly in situations where complete experimental stress-strain curve data are unavailable but standard mechanical test data are widely available. For future work, it is recommended that the model validation be extended by comparing the simulated curves with actual experimental stress-strain curve data, as well as exploring the application of the proposed approach to other material categories such as stainless steel and aluminum alloys.

AUTHORS' CONTRIBUTIONS

YP, NR, and **MN** contributed to the conceptualization of the study, methodology development, supervision, formal analysis, data validation, writing of the original draft, and reviewing and editing the manuscript. **HP, YP,** and **NR** were responsible for investigation, data curation, formal analysis, data visualization, and preparation of the original draft. Meanwhile, **MR, MN,** and **YP** contributed to supervision, project administration, and manuscript review and editing.

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